On Regularization, Formulation and Initialization of the Active Contour Models (Snakes)

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Abstract

In snake formulation, large regularization enhances the robustness against noise and incomplete data, while small values increase the accuracy in capturing boundary variations. We present a local minimax criterion which automatically determines the optimal regularization at every locations along the boundary with no added computation cost. We also modify existing energy formulations to repair deficiencies in internal energy and improve performance in external energy. This yields snakes that contain Hough transform as a special case. We can therefore initialize the snake efficiently and reliably using Hough transform.

1 Introduction

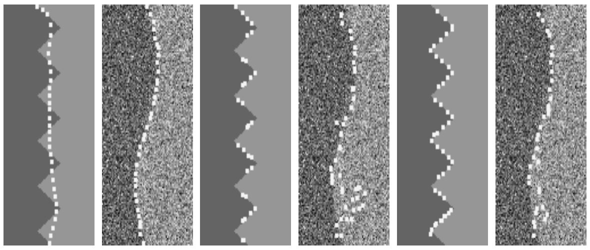
The active contour models (snakes) [1] turn boundary detection into an optimization problem. Given a snake $\mathbf{v} = \{v_i = (x_i, y_i) : i = 1, 2, \ldots, n\}$ one seeks to minimize its energy:

$$e(\mathbf{v}) = \sum_{i=1}^{n} \lambda_i E_{int}(v_i) + (1 - \lambda_i) E_{ext}(v_i) \qquad (1)$$

where the internal energy E_{int} imposes continuity and smoothness constraints; and the external energy E_{ext} attracts the snake to image features.

λ_i are the regularization parameters which significantly influence the solutions. Setting $\lambda \gg (1-\lambda)$ emphasizes regularization, yielding strongly model-driven solutions which are robust to noise. In contrast, small values of λ enable the snakes to effectively capture boundary discontinuities, but they also make the solutions very noise sensitive. Figure 1 illustrates these conflicting goals. Despite its significance, regularization selection were either ignored [1, 2, 3, 4], or based on empirical observations [5, 6] and heuristics [7].

Another factor which directly affects the solutions is energy formulation. The original E_{int} formulation [1] causes a snake to shrink around strong edge points. Ballooning force [6] and hard constraint on snaxel proximity [5] have been suggested, but these require prior information in setting the weighing factor or constraint. Moreover, existing curvature measure on E_{int} is not rotational and scale invariant, causing erroneous imposition of the smoothness constraint.



(a) $\lambda = 0.8$ (b) $\lambda = 0.2$ (c) minimax

Figure 1: Effects of Regularization. (a) Snake fails to capture corners but robust to noise; (b) Snake capture corners but noise sensitive; (c) good compromise.

Snaxel initialization is the next factor with significant impact. For satisfactory results, most existing methods [1, 5, 7] require human to judiciously place the initial snaxels near desired boundaries. Alternatively, short snakes are initialized at locations of high intensity gradient and they either overlap [2] or extend in length [3] to cover a longer boundary. However, the latter strategies cause the snake to lose the globalities that enable it to interpolate subjective contours across boundary gaps and noisy segments.

This paper tackles the fundamental issues on regularization, formulation and initialization of the active contour models. Section 1 presents the *local minimax criterion* which automatically determines the optimal regularization at every location along the boundary without adding computational cost. Section 2 proposes modifications in energy formulation to repair existing deficiencies in E_{int} and enhance performance in E_{ext}. Section 3 shows that Hough transform [8] is a special case of a snake and use it to initialize the snake efficiently from a global perspective. Section 4 presents some experimental results.

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2 Regularization

We begin by considering global regularization where λ is applied uniformly over an objective function with two terms E_{int} and E_{ext}. We seek $\mathbf{v}$ to minimize

$$e(\mathbf{v}, \lambda) = \lambda E_{int}(\mathbf{v}) + (1 - \lambda) E_{ext}(\mathbf{v}) \tag{2}$$

Suppose that in (2) the parameter λ is unknown because no prior information on the underlying variations on the boundary and the level of noise is available. Since values chosen *a priori* are unlikely to be satisfactory for all cases, it becomes necessary to define a criterion to determine λ. A viable criterion is to seek a $\mathbf{v}$ that minimizes, over all λ, the maximum of $e(\mathbf{v}, \lambda)$. This yields the *minimax* [9] criterion :

$$e(\mathbf{v}^*, \lambda^*) = \min_{\mathbf{v}} \max_{\lambda} \lambda E_{int}(\mathbf{v}) + (1 - \lambda) E_{ext}(\mathbf{v}) \tag{3}$$

where $\mathbf{v}^*$ and λ^* are, respectively, the minimax snake and regularization parameter.

As a function of λ, $e(\mathbf{v}, \lambda)$ is a straight line from $e(\mathbf{v}, 0) = E_{ext}(\mathbf{v})$ to $e(\mathbf{v}, 1) = E_{int}(\mathbf{v})$, as shown in Figure 2(a). Thus, for fixed $\mathbf{v}$, the maximum value of $e(\mathbf{v}, \lambda)$ occurs at either $\lambda = 0$ or $\lambda = 1$ and this maximum value is $\max(E_{int}(\mathbf{v}), E_{ext}(\mathbf{v}))$. For each $\lambda \in [0, 1]$, let $\mathbf{v}_\lambda$ be the *optimum snake* that minimizes equation (2) corresponding to the chosen λ, and let $U(\lambda) = e(\mathbf{v}_\lambda, \lambda)$. Note that since $\mathbf{v}_\lambda$ are optimum, $U(\lambda)$ is the minimum possible energy for λ. Moreover, $U(\lambda)$ is a continuous convex function regardless of the convexity of $E_{int}(\mathbf{v})$ and $E_{ext}(\mathbf{v})$.

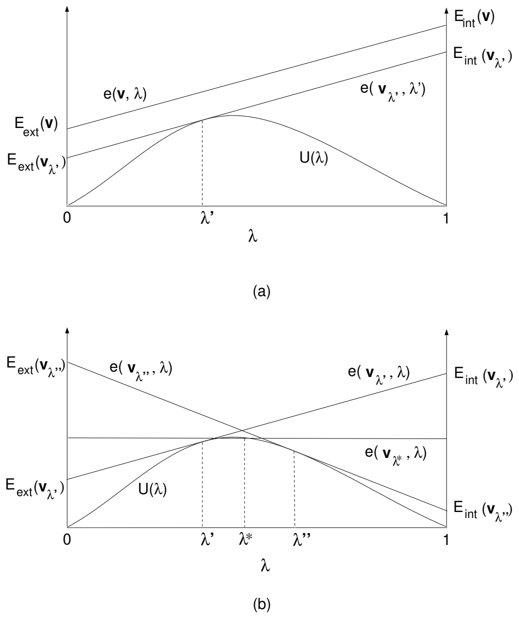


Figure 2: (a) Illustration of functions $e(\mathbf{v}, \lambda)$ and $U(\lambda)$; (b) Illustration of the minimax criterion when U has an interior maximum.

Suppose that $U(\lambda)$ assumes a typical form as depicted in Figure 2(a). The line labeled $e(\mathbf{v}, \lambda)$ is both parallel to $e(\mathbf{v}_{\lambda'}, \lambda')$ and tangent to $U(\lambda')$. Note that $\mathbf{v}$ cannot be a minimax solution because the line $e(\mathbf{v}, \lambda)$ lies completely above $e(\mathbf{v}_{\lambda'}, \lambda')$ and thus has a larger minimum value. Since $e(\mathbf{v}_{\lambda'}, \lambda')$ touches $U(\lambda)$ at $\lambda = \lambda'$, $\mathbf{v}_{\lambda'}$ is the optimum snake for parameter λ'. Because no snake can minimize (2) below $U(\lambda)$, given any $\mathbf{v}$, we can always find a tangent line that has the smallest $e(\mathbf{v}, \lambda)$. Consequently only an optimum snake can be the minimax solution for this figure. Moreover, by examination of Figure 2(b), we see that the minimax solution is an optimum snake $\mathbf{v}_{\lambda^*}$ for a parameter λ^* that maximizes $U(\lambda)$ over $\lambda \in [0, 1]$:

$$\min_{\mathbf{v}} \max_{\lambda} e(\mathbf{v}, \lambda) = \max_{\lambda} \min_{\mathbf{v}} e(\mathbf{v}, \lambda) \tag{4}$$

Since λ^* maximizes the minimum snake energy $U(\lambda)$, a minimax snake is the optimum snake that minimizes the worst case energy. Note that in this case it also equalizes the value of internal and external energy.

It can be shown that (4) is true even if $U(\lambda)$ does not assume the typical form *i.e.*, $\max_\lambda U(\lambda)$ occurs at $\lambda = 0$ or $\lambda = 1$.

Nonetheless a *global* regularization often causes over-smoothing of discontinuities in some parts of the boundary and excessive interpolation of noisy boundary in the rest, as evidenced in Figure 1. Hence we seek an alternate solution that will provide sufficient regularization to suppress the effect of noise while preserving boundary discontinuities, *i.e.*, , we wish to find $\mathbf{v}$ to minimize

$$e(\mathbf{v}, \Lambda) = \sum_{i=1}^{n} [\lambda_i E_{int}(v_i) + (1 - \lambda_i) E_{ext}(v_i)] \tag{5}$$

where $\Lambda = \{\lambda_1, \lambda_2, \ldots, \lambda_n\}$ are the unknown *local* regularization parameters. We apply the arguments presented above and seek a *local* minimax solution $\mathbf{v}^*$ that minimizes, over every λ_i, the maximum of $e(v_i, \lambda_i)$. This yields the *local minimax criterion* :

$$e(\mathbf{v}^*, \Lambda^*) = \min_{\mathbf{v}} \sum_{i=1}^{n} \max(E_{int}(v_i), E_{ext}(v_i)) \tag{6}$$

where $\mathbf{v}^*$ and Λ^* are the solutions. Note from (6) that this criterion completely bypasses the explicit selection of the *nuisance* parameters Λ^*. It can therefore be incorporated into discrete energy minimization algorithms [5, 7] with no added computational cost.

Finally, we generalize the arguments to cases with multiple constituent terms:

$$e(\mathbf{v}, \Lambda) = \sum_{i=1}^{n} e(v_i, \Lambda_i) = \sum_{i=1}^{n} \Lambda_i^T \mathbf{E}(v_i) \tag{7}$$

where $\mathbf{E}(v_i) = [E_1(v_i), E_2(v_i), \ldots, E_p(v_i)]^T$ and $\Lambda_i = [\lambda_{i1}, \lambda_{i2}, \ldots, \lambda_{ip}]^T$ are p-dimensional energy functionals and regularization parameters respectively. Let $\mathbf{1} = [1, 1, ..., 1]^T$. If Λ_i satisfies $\Lambda_i^T \mathbf{1} = 1$ for $i = 1, 2, \ldots, n$, then the local minimax criterion equalizes the energies and satisfies

$$e(\mathbf{v}^*, \Lambda^*) = \min_{\mathbf{v}} \sum_{i=1}^{n} \max_{j}(E_j(v_i)) \tag{8}$$

As energies are equalized at local minimax, the criterion requires each energy functional to be properly normalized. The next section presents normalized energy formulations which have also undergone some enhancements.

3 Energy Formulation

In general, E_{int} consists of a continuity term E_{cont} and a curvature term E_{curv} that give the snake rubber-sheet and thin-plate like behaviour respectively.

We define E_{cont} to measure the normalized deviation of the inter-snaxel distance from the averaged distance:

$$E_{cont}(i) = (d_\infty - \|v_i - v_{i-1}\|_\infty)^2 / d_\infty^2 \tag{9}$$

E_{cont} encourages snaxels to be evenly spaced and eliminates the tendency of shrinkage [1] and expansion [7] in other formulations. Normalizing the deviations with d_∞, the averaged inter-snaxel distance, makes E_{cont} scale invariant. Moreover, using L_∞ norm ($\|v_i\|_\infty = \max(|x_i|, |y_i|)$) corrects the bias to horizontal and vertical alignment that exists in the Euclidean norm (L_2), making E_{cont} rotational invariant.

For proper imposition of the smoothness constraint, E_{curv} should penalize bending in a manner which is scale and rotational invariant. This can be achieved by making E_{curv} a function of the *bending angle* ψ_i:

$$E_{curv}(i) = \frac{1}{2}(1 - \cos(\psi_i)) = \frac{1}{4}\left|\frac{u_i}{|u_i|} - \frac{u_{i-1}}{|u_{i-1}|}\right|^2 \tag{10}$$

The rightmost equality is obtained by using cosine rule and observing that $u_i = v_{i+1} - v_i$ as in Figure 3.

For external energy, we use both the magnitude and direction of intensity gradient to yield

$$E_{ext}(i) = 1 - \overline{|\nabla I(v_i)|}(1 - |\cos(\varphi_i)|) \tag{11}$$

where $\overline{|\nabla I(v_i)|}$ is the magnitude of the intensity gradient normalized to $[0, 1]$ and φ_i is the angle between the intensity gradient vector and the unit tangent vector at v_i.

Unlike methods [3] which use gradient direction to discard spurious edge points, we use the factor $1 - |\cos(\varphi_i)|$ to continuously weigh the gradient magnitude against any orientation inconsistency. Consequently, besides a non edge point, an edge point whose orientation disagrees with that of the overlaying snake may also yield large external energy. Hence the reformulated E_{ext} is able to discriminate against phantom lines while allowing legitimate corners to develop.

We compute the intensity gradient vectors by fitting planes in 2x2 windows using the method of least squares. Unlike methods [5, 6] that use complex edge detectors (*e.g.* Canny's), we do not accept or discard edges at this stage. We contend that these decisions are premature and thus defeat the purpose of regularization.

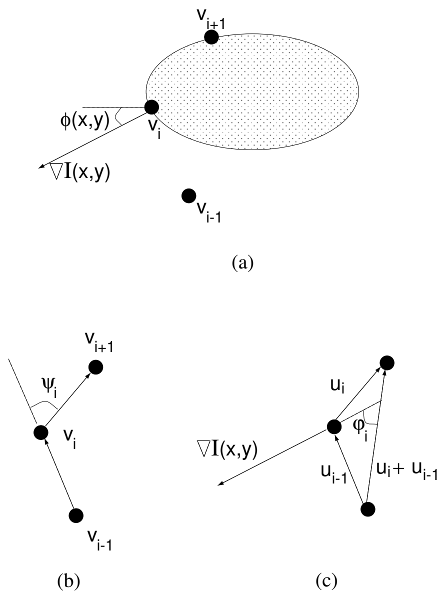


Figure 3: Snaxel and vector notations.

Finally, using the local minimax criterion, the energy to be minimized at snaxel v_i is simply:

$$e(i) = \max(E_{cont}(i), E_{curv}(i), E_{ext}(i)) \tag{12}$$

4 Initialization

It can be shown that when λ is sufficiently closed to 1 in (2), the optimum snake is a line for opened snake, and a circle for closed snake. Snakes that satisfy this structural constraints have constant E_{int} that can be ignored in energy minimization. Therefore

$$\mathbf{v}^* = \arg\min_{\mathbf{v}} E_{ext}(\mathbf{v}) = \arg\max_{\mathbf{v}} S_n(\mathbf{v}) \tag{13}$$

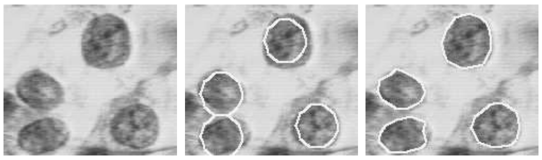
where $S_n(\mathbf{v}) = \sum_{i=1}^{n}\left[\overline{|\nabla I(x,y)|}(1 - |\cos(\varphi_i)|)\right]$. The problem is then equivalent to finding a line or circle that maximizes $S_n(\mathbf{v})$. This suggests a counting procedure that can be implemented using Hough transform.

$e(\mathbf{v}, \lambda)$ is continuous in λ because both E_{int} and E_{ext} are finite. Hence the optimum snake $\mathbf{v}^*_{\lambda_1}$ must be located near $\mathbf{v}^*_{\lambda_2}$ if the difference $|\lambda_1 - \lambda_2|$ is small. Since the initialized snake using Hough transform has $\lambda \approx 1$, we can bring the snaxels into nearby optimal positions by relaxing the regularization parameters to their minimax values. Because parameter selection is implicit under the local minimax criterion, this can simply be achieved through energy minimization.

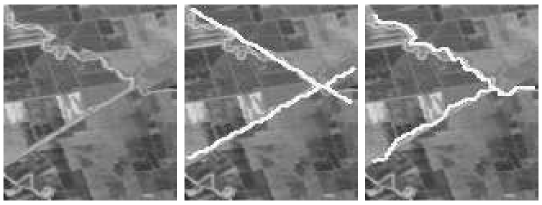
This initialization strategy exploits the efficiency and globality of Hough transform in the presence of noise and boundary gaps. Moreover, since the snaxels are initialized near optimal positions, only small search windows are needed in energy minimization.

5 Experiments

Figure 4 shows results obtained by applying the proposed realization on an image of body cells and an aerial photograph. After initializing with Hough transform, regularization was selected implicitly during energy minimization to obtain the final solutions.



(a)



(b)

Figure 4: Initial and Final Positions

In addition, we examine the effect of regularization using synthesized sawtooth image as in Figure 1. We compute the mean squared error between the ideal boundary to final boundaries obtained using $\lambda = 0.2, 0.4, 0.6, 0.8$ and local minimax. Using image contrast of 1, we add zero mean gaussian white noise and plot the MSE against the noise variance.

Figure 5 shows a typical plot from these experiments. In general, snakes with large λ are resistive to bending, and thus yield large errors for sharp sawtooth angles. However, they also suffer less severe degradation with higher noise. In contrast, small λ capture the corners more accurately when noise is low, but degrades rapidly with noise due to lack of regularization. The minimax criterion exhibits good compromise (see also Figure 1), providing the capability to capture boundary discontinuities but only suffer moderate degradation with noise.

Energy minimization using dynamic programming [5] guarantees convergence, but suffers dramatic speed degradation as one increases the size of search windows. Point-wise method [7] improves speed performance but does not guarantee convergence. For practicality, we use a modified greedy algorithm which also considers the energy of the two neighboring snaxels.

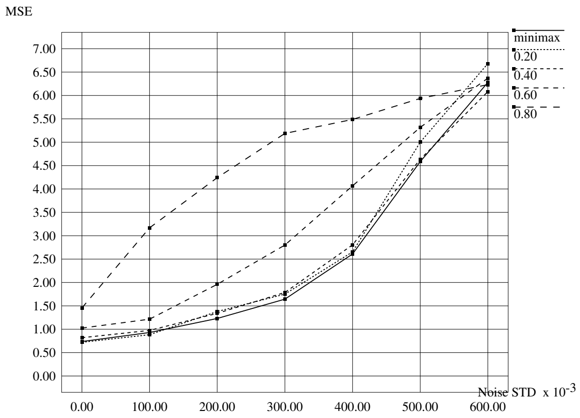


Figure 5: Plots of MSE against noise variance

This algorithm moves a snaxel v_i to a location that minimizes the sum $e(i-1) + e(i) + e(i+1)$, rather than $e(i)$ as in [7]. This yields monotonically decreasing energy that ensures convergence.

6 Conclusions

We treated the fundamental issues on regularization, formulation and initialization of the active contour models. The experimental results confirmed the validity of our theoretical analysis.

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