ME 748: Optimum Design of Mechanical Elements and Systems
Algorithms for Unconstrained Optimization

Problem: $\min_{\bar{x}=\{x_1,x_2,\ldots,x_N\}} f(\bar{x})$; Special case quadratic $f(\bar{x}) = \frac{1}{2}\bar{x}^T Q\bar{x} + \bar{x}^T b + c$; Q is assumed to be positive definite

	Algorithm	Description	Performance		Comments
			Quadratic	**General**	
0th Order	*Grid*	Sample $\bar{x}$ over a grid (first coarse and then fine)	Robust, and easy to code but extremely slow for large N, where N is the number of free variables in $\bar{x}$		Recommended as the starting method for global optimization
	Random	Random sample of $\bar{x}$ (first coarse and then fine)	Robust, and easy to code but extremely slow for large N, where N is the number of free variables in $\bar{x}$		
	Alternating Coordinate	Perform a line-search by cycling among the unit vectors e_i	Converges in N line-searches if Q is diagonal. Else can be very slow.	Converges in N line-searches if the Hessian is diagonal. Else can be very slow.	
	Powell	First perform a line-search by cycling among the unit vectors e_i, then search along newly generated directions	Converges in N line-searches if Q is diagonal. Else quadratic convergence.	Converges in N line-searches if Hessian is diagonal. Else quadratic convergence.	Recommended as the best overall among 0th order methods.
1st Order	*Steepest Descent*	Perform a line-search along the gradient $\nabla f(\bar{x})$ at each point.	Converges in 1 line-search if Q has identical eigen-values. Else poor performance	Typically poor performance	
	Fletcher Reeves Conjugate Gradient	Perform a line-search along conjugate directions	Converges in N line-searches.	Typically excellent performance	One of the best among 1st order
2nd Order	*Newton*	Generate next point using Hessian and gradient	Converges in 1 step.	May converge to maxima or saddle point or … Need line-search to hit minima.	Expensive to compute $\left(\nabla^2 f(\bar{x})\right)^{-1}$
	BFGS Quasi-Newton	Generate next point using approximate Hessian and gradient	Cubic convergence	Excellent performance.	Most popular