CS810: Homework 3 Due date: Tuesday , April 15th, 2003

1. Given a Boolean function f on n variables, define a *minterm* of f as a conjunction c of several literals $c = \hat{x_{i_1}} \wedge \ldots \wedge \hat{x_{i_k}}$, where each $\hat{x}$ = either x or $\bar{x}$, such that, a partial assignment according to c satisfies f, and furthermore, no proper subassignment will satisfy f.

   (Here a partial assignment according to c, means as follows: each literal which is an unnegated variable is assigned 1 (True), and each negated variable is assigned 0 (False).) e.g., suppose $c = x_2 \wedge \bar{x_5} \wedge x_9$, then the partial assignment according to this c sets $x_2 = 1$ and $x_5 = 0$ and $x_9 = 1$. A proper subassignment assigns a proper subset of these literals in c. e.g., for the above c, the partial assignment $x_2 = 1$ and $x_5 = 0$ is a proper subassignment.)

   In short, a minterm is a minimal assignment that can make f true. e.g., for the majority function on $2k - 1$ variables, there are exactly $\binom{2k-1}{k}$ many minterms. What are they?

   Replace the notion of Decision tree depth DC as we did in class by minterm size. Carry out the proof of the main switching lemma in terms of this notion of minterm size. Your theorem should read something like this: For any t-AND-OR circuit, if we assign it with a random restriction ρ with prob of $*$ equal to some p, then for any $\Delta \geq 0$, with probability less than α^Δ the function after the restriction has any minterm of size $\geq \Delta$. (Here α should be something depend on p and t.)

2. Define a *maxterm* as a minimal assignment that forces the function f to be false. What's the relationship of maxterm of f and minterm of $\bar{f}$.

3. Show that if any function has $DC \leq t$, then f can be expressed as both a t-OR-AND as well as a t-AND-OR.

4. Show that if all minterms of f are of size $\leq t$, then f can be expressed as a t-OR-AND.

   But not conversely: Consider $f = (z \wedge x_1 \wedge \ldots \wedge x_n) \vee (\bar{z} \wedge y_1 \wedge \ldots \wedge y_n)$. f is expressible as an $(n+1)$-OR-AND, but not all minterms of f are of size $\leq n + 1$.

   Which minterm? Prove it is a minterm.

5. If both f and $\bar{f}$ can be expressed as a t-OR-AND (ie. f can be expressed both as a t-OR-AND as well as a t-AND-OR), then $DC(f) \leq t^2$.