SPRING 1993
COMPUTER SCIENCES DEPARTMENT
UNIVERSITY OF WISCONSIN—MADISON
PH. D. QUALIFYING EXAMINATION

Modeling and Analysis
Depth Examination

Wednesday, February 10, 1993
3:00 – 7:00 PM
113 Psychology

GENERAL INSTRUCTIONS:

1. Answer each question in a separate book.
2. Indicate on the cover of *each* book the area of the exam, your code number, and the question answered in that book. On *one* of your books list the numbers of *all* the questions answered. *Do not write your name on any answer book.*
3. Return all answer books in the folder provided. Additional answer books are available if needed.

SPECIFIC INSTRUCTIONS:

Answer *all seven* questions.

POLICY ON MISPRINTS AND AMBIGUITIES:

The Exam Committee tires to proofread the exam as carefully as possible. Nevertheless, the exam sometimes contains misprints and ambiguities. If you are convinced a problem has been stated incorrectly, mention this to the proctor. If necessary, the proctor can contact a representative of the area to resolve problems during the *first hour* of the exam. In any case, you should indicate your interpretation of the problem in your written answer. Your interpretation should be such that the problem is nontrivial.

MA Depth Screening Exam

Spring 1993

Directions: Answer ALL of the following questions. Show your work.

1. Short Answer Questions:

   (a) Let R_i and R_{ii} denote the mean response time for system (i) and (ii) below, respectively. What is the relationship between R_i and R_{ii}?

      (i) An M/M/1 system with arrival rate λ and mean service time S.

      (ii) An M/M/1 system with arrival rate 2λ and mean service time $\frac{S}{2}$.

   (b) Which random variable has a higher coefficient of variation?

      (i) X, an exponentially distributed r.v. with parameter μ, or

      (ii) Y, where Y = X + 1

   (c) For the discrete-parameter Markov chain shown in the Figure below, what is the steady state probability that the system is in state 0?

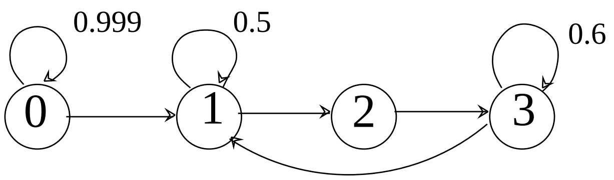

Figure 1:

2. Suppose we have a system A that is accurately modeled by a product-form queueing network. Now suppose we upgrade the system so that the mean service time at every resource is reduced by 50%. Call the upgraded system B and assume it is also accurately modeled by the product-form queueing network with the corresponding changes in service time parameters.

   Use either the product-form equations for the network state probabilities or the mean value analysis (mva) equations for the network to explain what happens quantitatively to the resource utilizations, mean queue length at each resource, network throughput, and network mean response time when the system is upgraded, for each of the following cases:

   (a) The queueing network is a closed, single-class network.

   (b) The queueing network is an open, single-class network.

3. Consider the following M/M/1 FCFS queue with impatient customers. Customers arrive with rate λ and their service demand is exponentially distributed with parameter μ. If an arriving customer does not receive service before an exponentially distributed amount of time with parameter α, the customer simply leaves the queue.
   (a) Draw the Markov Chain for the system.
   (b) Under what conditions is this queue stable?

4. A number of transactions access pages from a common buffer pool that contains a total of 100 pages. Each transaction executes on its own server for a random amount of time with mean 10 milliseconds and then makes a page request. If no other transaction is accessing the page, the transaction holds the page for an average of 200 milliseconds. Otherwise the transaction waits in a FCFS queue for the page. After completing its page access the transaction releases the page and returns to its processing phase. Suppose that 80% of the page requests are uniformly spread across pages 1 through 20 (i.e., the *hot* pages) and the remaining 20% of the requests are uniformly distributed across all 100 pages. What is the maximum possible throughput of the system?

5. Two processes with i.i.d. execution times X_1 and X_2 synchronize at a barrier. That is, the first process to complete execution waits for its partner to reach the barrier. The barrier completes at the time that the second process completes its execution. Suppose that X_1 and X_2 each have a uniform(0,1) distribution.
   (a) If the processes execute in parallel and both begin execution at the same time, what is the mean time until barrier completion?
   (b) Suppose that instead of one barrier synchronization, the processes undergo N barrier synchronizations, where N is a geometric random variable with parameter p. What is the mean time for all N barriers to complete?
   (c) What is the answer to question (a) if X_1 and X_2 are each uniform(85,115)?

6. A random variable X has a Generalized Exponential (GE) distribution if

$$X = \begin{cases} \text{Exponential}(\mu_1) & \text{with probability } \alpha_1 \\ 0 & \text{otherwise} \end{cases}$$

   (a) Express $\overline{X}$, $\overline{X^2}$, and C_X in terms of μ_1 and α_1.
   (b) Find the mean response time of an M/GE/1 queue with arrival rate $\lambda < 1/\overline{X}$ for
      (i) the FCFS scheduling discipline
      (ii) the PS scheduling discipline

      For which discipline is the mean response time greater?
   (c) Does the M/GE/1 FCFS queue satisfy local balance? Explain your answer.

7. How would you solve for the mean response time in the queueing network shown below, assuming that service times at all centers are exponentially distributed? (At the queue labeled "random", scheduling is non-preemptive and at each service completion, each customer in the queue is equally likely to be served next.)

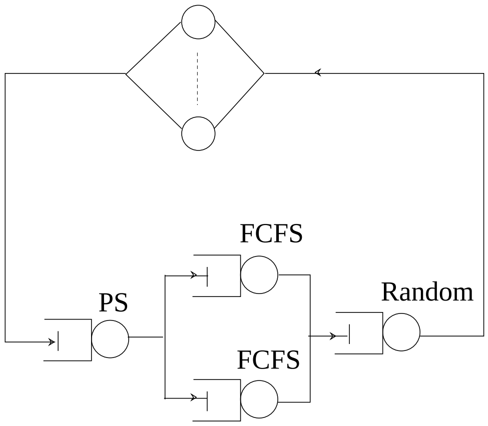

Figure 2: